Exercise 2.6.1

Explain this paradox: a simple harmonic oscillator $m\ddot{x} = -kx$ is a system that oscillates in one dimension (along the x-axis). But the text says one-dimensional systems can't oscillate.

Solution

There is no paradox. According to the text, "there are no periodic solutions to $\dot{x} = f(x)$." The equation here is different.

$$\ddot{x} = -\frac{k}{m}x = f(x) \tag{1}$$

If the substitution $v = \dot{x}$ is made, then equation (1) becomes $\dot{v} = -(k/m)x$.

$$\begin{cases} \dot{x} = v \\ \dot{v} = -\dfrac{k}{m}x \end{cases}$$

This system of first-order ODEs is two-dimensional because x and v are present and need to be solved for. Therefore, the simple harmonic oscillator can oscillate.

The reason there are no periodic solutions to

$$\{\dot{x} = f(x)$$

is because this system of first-order ODEs is one-dimensional—only x is present.